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THE NEW BRUNSWICK MASONRY DAM.

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To construct an embankment in order to store up water, seems to many, who have had little or no experience in engineering, to be the simplest of all constructions and demands very little educated judgment.

This fact may possibly be the cause of the failures of many reservoir embankments which has led to a great sacrifice of capital, and in some cases to the death of many human beings. Among the recent failures may be mentioned the Bradfield, England, water-works, in 1864, and in our own country the Hartford, Conn., water-works, in 1866, and the Worcester, Mass., water-works, in 1875.

An instance of less importance, but of greater interest to us, is the failure of the dam of the New Brunswick water-works, at Weston's Mills. It failed at 6:30 P. M., February 25th, 1888. At the time of its failure the dam was fifteen feet high and about 225 feet long. It had for a long time been regarded as unsafe by the Superintendent. It is said that a dam has existed at this point since 1743; at all events, it was built ten feet high prior to 1800.

When the New Brunswick Water-Works Department obtained possession they increased its height to eleven feet. In 1884 it was raised by four feet and capped with sand-stone blocks.

A depth of two feet of water upon its crest has not been unusual in the past, and water has been known to rise three feet above the crest.

The Board of Water Commissioners accepted the plans for a new dam, as drawn by Josiah Tice, City Surveyor of New Brunswick, and the contract for the masonry work was given to Shanley & Co., of Newark, N. J. The plans and specifications are as follows:

The work is to consist of a dam, abutment, pier and two buttresses, all of which are represented in the draughtings. The dam is to be of masonry, 160 feet long, resting on a foundation which is to be constructed of heavy beton to a point 2 feet below the top of the existing apron of the old dam. The new dam is to be set back 2 feet from the front line of the foundation. It is to be 15 feet high and 9½ feet thick at its base. It is to have on the face a batter of 6 inches in the

15 feet rise, and a total batter on the rear of 7 feet. The whole dam is to be topped with a coping of blue-stone at least 6 inches thick, having a slope toward the pond of 1 foot in 3, and dowelled to the masonry. The pier which separates the dam from the flood-gates will be 6 feet thick and $8\frac{1}{2}$ feet wide, and will extend 2 feet in front of the dam to the line of the foundation. The abutment rises 2 feet above the dam.

The foundation is to be placed on solid rock, having a pitch toward the pond of $4\frac{1}{2}$ inches. Horizontal beds are to be cut $11\frac{1}{2}$ feet wide, having a pitch toward the pond of $4\frac{1}{2}$ inches. The foundation will consist of two outer walls, laid in good Portland cement, after the manner of headers and stretchers; headers are not to be less than $3\frac{1}{2}$ feet in length, and two feet in width, with a rise of at least half a foot.

The stretchers are not to be less than 2 feet wide.

The stones may be laid irregular as to coursing, but must have a good bond. No dressing will be required in the foundation except for the first layer, which must be dressed to a good bed. The space between the walls is to be filled with alternate layers of beton and rubble.

The masonry in the dam proper will be of first-class coursed rubble, and laid in the best quality of Portland cement.

The stones are to be laid after the manner of headers and stretchers in even courses.

The question has arisen as to whether a dam built according to the above plans and specifications would be stable enough to resist the water-pressure.

The pressure exerted by a fluid in any direction upon a surface is equal to the weight of a column of the fluid whose base is the projection of the surface at right angles to the given direction, and whose height is the depth of the center of gravity of the surface below the surface of the fluid. (Bowser's Hydromechanics.)

Let $P =$ the pressure.

$x =$ the projection of the surface at right angles to the given direction.

$a =$ the depth of the center of gravity below the surface of the fluid.

$w =$ the weight of a cubic foot of water.

Then the formula will be $P = x\,a\,w$.

In this case $x = 21.5$ feet, $a = (\frac{21.5}{2} + 3)$ feet, there being 3 feet of water on the crest; $w = 62\frac{1}{2}$ lbs. Substituting in the formula gives $P = 21.5\ [\frac{1}{2}\ (21.5) + 3]\ 62.5$.

Therefore $P = 18476$ lbs.

The formula for finding the point of application of pressure as given in Bowser's Hydromechanics, is

$$\bar{x} = \tfrac{2}{3}\,\frac{a^3 - b^3}{a^2 - b^2}$$ in which $\bar{x} =$ is

the point of application measured from the surface of the fluid. a and b are the distances of the bottom and top of the dam from the surface of the fluid.

In the above case $a = 24.5$ feet, $b = 3$ feet. Substituting in formula

$$\bar{x} = \tfrac{2}{3}\,\frac{a^3 - b^3}{a^2 - b^2} = \tfrac{2}{3}\,\frac{(24.5)^3 - (3)^3}{(24.5)^2 - (3)^2} = \tfrac{2}{3}\,\frac{24706 - 27}{600.25 - 9} = 16.6 \text{ feet.}$$

Therefore $a - \bar{x} = 7.9$ feet.

Measuring this distance on line from base gives (x^1) as the point of application.

In flood flow the water backs up on the dam to 7.5 feet on the masonry. This gives a reverse pressure, which acts in opposition to the direct horizontal pressure and tends to give stability to the dam. For finding this reverse pressure, the formula is

$P = x\,a\,w.$ $x = 14$ feet, $a = 7$ feet, $w = 62\tfrac{1}{2}$ lbs.

Substituting gives, $P = 14 \times 7 \times 62\tfrac{1}{2} = 6125$ lbs.

In this case $b = o$. Therefore formula for application becomes, $\bar{x} = \tfrac{2}{3}a$. Therefore $a - \bar{x} = \tfrac{1}{3}a$; $a = 14$.

$a - \bar{x} = 4.666$ feet.

Therefore the reverse pressure acts at the point 4.666 above the base. It is necessary to find what force this pressure is equal to acting at the same point as the direct horizontal pressure. Let $Z =$ this force and proportioning it to the arm gives, $7.9\,Z = 6125 \times 4.666$.

$Z = 3617$ lbs.

Subtracting this reverse force from the direct horizontal force gives the final horizontal force.

$P = 18476 - 3617 = 14859$ lbs.

In computing the weight of the foundation, masonry and water resting upon it, we take the weight of the foundation as 125 lbs. to the cubic foot, the weight of the masonry as 150 lbs. to the cubic foot, and that of the water at $62\tfrac{1}{2}$ lbs. to the cubic foot.

Then in each case taking a section 1 foot wide:

The weight of section of foundation $= 6.5 \times 11.5 \times 150 = 9344.$
The weight of section of 1st course $= 9.5 \times 1.5 \times 150 = 2137.$
The weight of section of 2d course $= 8.75 \times 1.5 \times 150 = 1969.$
The weight of section of 3d course $= 7.25 \times 1.5 \times 150 = 1800.$
The weight of section of 4th course $= 6.5 \times 1.5 \times 150 = 1631.$
The weight of section of 5th course $= 5.75 \times 1.5 \times 150 = 1462.$
The weight of section of 6th course $= 5 \times 1.5 \times 150 = 1294.$
The weight of section of 7th course $= 4.5 \times 1.25 \times 150 = 1125.$
The weight of section of 8th course $= 4.5 \times 1.25 \times 150 = 844.$
The weight of section of 9th course $= 4 \times 1.25 \times 150 = 750.$
The weight of section of 10th course $= 3.5\ [\tfrac{1}{2}(2 + .8)]\ 150 = 735.$

Giving for the total weight of a section of dam $= 23091$ lbs. The weight of the water resting on the dam is found by adding the weights of water resting on ab, cd, etc. Fig. No. 3.

Weight of water resting on $ab = .75 \times 16.5 \times 62.5 = 773$ lbs.
Weight of water resting on $cd = .75 \times 15 \times 62.5 = 703$ lbs.
Weight of water resting on $ef = .75 \times 13.5 \times 62.5 = 633$ lbs.
Weight of water resting on $gh = .75 \times 12 \times 62.5 = 562$ lbs.
Weight of water resting on $ik = .75 \times 10\ 5 \times 62.5 = 492$ lbs.
Weight of water resting on $em = .75 \times 9 \times 62.5 = 422$ lbs.
Weight of water resting on $no = .75 \times 7.5 \times 62.5 = 352$ lbs.
Weight of water resting on $pr = .5 \times 6.25 \times 62.5 = 195$ lbs.
Weight of water resting on $st = .5 \times 5 \times 62.5 = 156$ lbs.
Weight of water resting on top of dam $= 3.8[\frac{1}{2}(4.3+3)]62.5 = 867.$
Total weight of water resting on section $= 5155$ lbs.

The weight of the section of the dam, plus the weight of the water resting upon it, must be capable of resisting the horizontal pressure of the water.

The weight of the dam and the weight of the water resting on it act downward through the center of gravity of their mass.

The center of gravity of the section of the dam is found by the method of moments, first by finding the center of gravity of these courses taken together with the third course, and so on.

The distance between the center of gravity of the 1st and 2d is equal to 1.5 feet. Weight of the 1st course = 735 lbs. The weight of the 2d course = 750 lbs. Letting $x =$ the distance from the center of gravity of the second course to the required center of gravity, we have then

$$750\,x = 735\,(1.5 - x).$$
$$1485\,x = 1125.$$
$$x = .74 \text{ feet.}$$

Measuring this distance on the line (at Fig. 1) gives the required center of gravity at (1).

The weight of the 1st and 2d courses equals 1485 lbs. The weight of the 3d course = 843 lbs. Therefore to find the center of gravity of the 1st and 2d and 3d courses, let $x =$ the distance of the required center of gravity from (1). Then

$$1485\,x = 844\,(2 - x).$$
$$2329\,x = 1688.$$
$$x = .72 \text{ feet.}$$

Laying this distance off on the line, 1, Fig. (1), gives the required center of gravity at (2).

The distance between the center of gravity of the 1st, 2d and 3d, and the center of gravity of the 4th course $= 2\frac{5}{8}$ feet.

Let $x =$ the distance from (2) to the center of gravity of the 1st, 2d, 3d, and 4th, courses. Weight of the 1st, 2d, and 3d = 2329, the weight of the 4th course = 1125, then,

$$2329\,x = 1125\,(2\tfrac{5}{8} - x) \text{ or, } x = .92 \text{ feet.}$$

Laying off this distance on the line (2d) gives center of gravity at (3).

Proceeding in like manner, the distance between the center of gravity of the 1st, 2d, 3d and 4th, and that of the 5th course = 3.4 feet.

Therefore 3454 $x = 1294\,(3.4 - x)$.

$x = .92$ feet.

Laying off this distance on the line (3d) gives the required center of gravity at (4).

The distance between the center of gravity of the 1st, 2d, 3d, 4th, 5th courses and that of the 6th course = 4 feet.

Weight of the 1st, 2d, 3d, 4th and 5th courses = 4748 lbs.

Weight of the 6th course = 1462 lbs.

Therefore 4748 $x = 1462\,(4 - x)$.

6210 $x = 5848$.

$x = .94$ feet.

This gives the required center of gravity at (5).

The distance between the center of gravity of the 1st, 2d, 3d, 4th, 5th, 6th courses and that of the 7th course = 4.6 feet.

The weight of the 1st, 2d, 3d, 4th, 5th, 6th courses = 6210 lbs.

The weight of the 7th course = 1631 lbs.

Therefore 6210 $x = 1631\,(4.6 - x)$.

$x = .94$ feet.

Lay this distance off on the line (5 *g*) gives the required center of gravity at (6).

The distance between the center of gravity of the 1st, 2d, 3d, 4th, 5th, 6th, 7th courses and the 8th = 5.2 feet.

Weight of 1st, 2d, 3d, etc., courses = 7841 lbs.

Weight of 8th course = 1800 lbs.

Therefore, 7841 $x = 1800\,(5.2 - x)$.

$x = .97$ feet.

Measuring this distance off on line (6 *h*) gives the required center of gravity at (7).

The distance between the center of gravity of the 1st, 2d, 3d, 4th, 5th, 6th, 7th, 8th courses and that of the 9th course = 5.75 feet.

The weight of the 1st, 2d, 3d, 4th, etc., courses = 9641 lbs.

The weight of the 9th course = 1969 lbs.

Therefore 9641 $x = 1969\,(5.75 - x)$.

$x = .98$ feet.

Measuring this distance off on the line (7 *k*) gave the required center of gravity at (8).

The distance between the center of gravity of 1st, 2d, 3d, ... 9th courses and the 10th course = 6 feet.

Weight of the 1st, 2d, ... 9th courses = 11610 lbs.

Weight of the 10th course = 2137 lbs.

Therefore 11610 $x = 2137\,(6 - x)$.

$x = .99$ feet.

Measuring this distance off on line (8 *l*) gives the required center of gravity at (9), which is the center of gravity of the masonry on the foundation.

The center of gravity of the masonry and the foundation is found by the same method. The distance from the center of the masonry to the center of gravity of the foundation is = 9.1 feet.

The weight of the foundation = 9344 lbs.

The weight of the masonry = 13747 lbs.

Therefore $13747\,x = 9344\,(9.1 - x)$.

$x = 3.68$ feet.

Measuring this distance off on the line joining centers of gravity gives the center of gravity of masonry and foundation at (10), Fig. 3.

The center of gravity of the water resting on the masonry was found in the same manner.

The distance between the center of the water resting on the top of the dam and that of the water resting on *ts*, Fig. No. 2, is = 2.1 feet.

Weight of the water on top of the dam = 867 lbs.

Weight of the water on *ts* = 156 lbs.

Therefore $867\,x = 156\,(2.1 - x)$.

$x = .32$ feet.

Measuring this distance off on the line (*ja*), Fig. 2, gives the required center of gravity at (1).

The distance between the center of gravity of the 1st and 2d columns of water and that of the 3d column of water = 2.6 feet.

The weight of the 1st and 2d columns = 1023 lbs.

The weight of the 3d column = 195 lbs.

Therefore $1023\,x = 195\,(2.6 - x)$.

$x = .42$ feet, which measured off on the line (1 *b*) gives the center at (2).

The distance between the center of gravity of the 1st, 2d, 3d columns and that of the fourth is = 3.6 feet.

Weight of the 1st, 2d, 3d columns = 1218 lbs.

Weight of the 4th column = 352 lbs.

Therefore $1218\,x = 352\,(3.6 - x)$.

$x = .8$ feet—which measured off on the line ($2\,c^1$) gives the required center of gravity at (3).

The distance between the center of gravity of the 1st, 2d, 3d, 4th columns and that of the 5th = 3.2 feet.

The weight of the 1st, 2d, 3d, 4th, columns = 1570 lbs.

The weight of the 5th column = 422 lbs.

Therefore $1570\,x = 422\,(3.2 - x)$.

$x = .68$ feet—which measured off on the line ($3\,d^1$) gives the required center of gravity at (4).

The distance between the center of gravity of the 1st, 2d, . . . 5th columns and that of the 6th column = 3.4 feet.

The weight of the 1st 2d, . . . 5th columns = 1992 lbs.

The weight of the 6th column = 492 lbs.

Therefore $1992\,x = 492\,(3.4 - x)$.

$x = .67$ feet—which measured on the line ($4\,e^1$) gives the required center of gravity at (5).

The distance between the center of gravity of 1st, 2d, . . . 6th columns and that of the 7th column = 3.7 feet.

The weight of the 1st, 2d, . . . 6th columns = 2484 lbs.

The weight of the 7th column = 562 lbs.

Therefore $2484\,x = 562\,(3.7 - x)$.

$x = .68$ feet—which measured off on the line ($5\,f^1$) gives the required center of gravity at (6).

The distance between the center of gravity of the 1st, 2d, ... 7th columns and that of the 8th column = 4.1 feet.

The weight of the 1st, 2d, ... 7th columns = 3046 lbs.

The weight of the 8th column = 633 lbs.

Therefore $3046\,x = 633\,(4.1 - x)$.

$x = 7.4$ feet—which measured off on the line ($6\,g^1$) gives the center of gravity at (7).

The distance between the center of gravity of the 1st, 2d, ... 8th columns of water and that of the 9th = 4.4 feet.

The weight of the 1st, 2d, ... 8th columns = 3679 lbs.

The weight of the 9th column = 703 lbs.

Therefore $3679\,x = 703\,(4.4 - x)$.

$x = .7$ feet—which measured off on the line ($7\,h^1$) gives the required center of gravity at (8).

The distance between the center of gravity of the 1st, 2d, ... 9th columns and that of the 10th column = 4.8.

The weight of the 1st, 2d . . . . 9th columns = 4382 lbs.

The weight of the 10th column = 773 lbs.

Therefore $4382\,x = 773\,(4.8 - x)$.

$x = .72$ feet—which measured off on the line ($8\,k^1$) gives the center of gravity of the water resting on the section of the dam at (9).

Since the weight of the masonry, the foundation and the water resting on the dam act together to resist the final horizontal pressure, we must find the center of gravity of their mass. This is done by method of moments.

The distance between the center of gravity of the section of dam and the center of gravity of the water resting on the section is = 12.5 feet.

The weight of the section of the dam = 23091 lbs.

The weight of the water resting on the section = 5155 lbs.

Therefore $23091\,x = 5155\,(12.5 - x)$.

$x = 2.21$ feet.

Measuring this distance off on the line *a* 9, Fig. (3), gives the center of gravity at (9^1).

The effect of a pressure when applied to a solid body is the same at whatever point in the line of its direction it is applied; therefore we may consider both the weight of the whole mass and the horizontal pressure as acting at (x^1), Fig. No. 3. Lay off, by scale, on the vertical through x^1, a distance to represent the weight of the whole mass and on the horizontal, through the same point, a distance to represent the horizontal pressure. The resultant of these forces will be the line of pressure and should cut the base within its center third. Completing the parallelogram, the diagonal ($x\ x^1$) cuts the base at (x), Fig. 3, a distance of 1.8 feet from the middle third of the base.

In the same manner we find the points in which the line of pressure cuts the base at the different joints above the foundation.

For the horizontal pressure of the water against the dam above the joint (a^1), Fig. No. 3, we have the formula,

$$P = x\ a\ w. \quad x = 15 \text{ feet}; \ a = 10\tfrac{1}{2} \text{ feet}; \ w = 62\tfrac{1}{2} \text{ lbs.}$$

Therefore $P = 9843$ lbs.

For the point of application we have, $\bar{x} = \frac{2}{3}\,\frac{a^3 - b^3}{a^2 - b^2}$. $a = 18$ feet; $b = 3$ feet.

Substituting gives $\bar{x} = \frac{2}{3}\,\frac{18^3 - 3^3}{18^2 - 3^2} = 12.28$ feet.

$a - \bar{x} = 18 - 12.28 = 5.72$ feet, which measured on the line from the base upwards gives the point of application to be at (y^1).

For the reverse pressure on this part of the dam we have the formula, $P = x\ a\ w$. $x = 7.5$ feet; $a = 3.75$ feet; $w = 62.5$ lbs.

Therefore $P = 7.5 \times 3.75 \times 62.5 = 1757$ lbs.

This pressure acts at $\frac{1}{3}$ the height of the water $= 2.5$ feet. Proportion this pressure to the same arm into which the direct pressure acts. Let $Z =$ to this required pressure.

Then $5.72\ Z = 1757 \times 2.5$.

$Z = 768$ lbs.

Subtracting this pressure from the direct horizontal pressure, we have $9843 - 768 = 9075$ lbs. This is the final horizontal pressure.

The weight of the masonry and the weight of the water resting upon it is $= 18902$ lbs. The center of gravity of the masonry and the water resting upon it is found by method of moments. The distance between the center of gravity $= 7.3$ feet.

Therefore $3747\ x = 5155\ (7.3 - x)$.

$x = 1.9$ feet, which measured off on the line (a_5 8) gives the center of gravity of the masonry and water to be at (8^1), Fig. 3.

Lay off, by scale, on the vertical line through (y^1); and horizontally the force is 9075 lbs. Completing the parallelogram gives the resultant line of pressure which cuts the base at (y), Fig. No. 3, a distance of 1.6 feet outside of the middle third.

To find the point in which the resultant pressure cuts the base at joint (b^1), Fig. 3. The formula for the horizontal pressure is,

$$P = x\ a\ w. \quad x = 13.5 \text{ feet}; \ a = 9.75 \text{ feet}; \ w = 62.5 \text{ lbs.}$$

Therefore $P = 13.5 \times 9.75 \times 62.5 = 8226$ lbs.

For the point of application, $\bar{x} = \frac{2}{3} \frac{a^3 - b^3}{a^2 - b^2}$. $a = 16.5$ feet; $b = 3$ feet.

Substituting gives $\bar{x} = \frac{2}{3} \frac{(16.5)^3 - (3)^3}{16.5^2 - (3)^2} = 11.3$ feet.

$a - \bar{x} = 5.2$ feet. Measuring this distance off on line ( ) gives the point of application.

The formula for the reverse pressure is,

$$P = x\,a\,w. \quad x = 6;\ a = 3;\ w = 62.5 \text{ lbs.}$$

Therefore $P = 6 \times 3 \times 62.5 = 1125$ lbs.

The point of this pressure is at $\frac{1}{3}$ the height of the water, or 2 feet from the base at (b^1), Fig. No. 3. Proportioning this pressure to the arm of direct horizontal force, 5.2 feet, we have:

$$5.2\ Z = 1125 \times 2.$$
$$Z = 433 \text{ lbs.}$$

The final horizontal force $= 8226 - 433 = 7793$ lbs. The weight of the masonry and the water in this case $= 15992$ lbs.

The center of gravity of masonry and water is found by method of moments.

The distance between their centers of gravity $= 6.65$ feet.

Therefore $11610\,x = 4382\,(6.65 - x)$.

$$x = 1.82 \text{ feet.}$$

Measuring this distance off on the line ($8\ a_5$) gives this center of gravity at (8^1).

Laying off on a vertical through Z^1, Fig. 3, the weight 15992 lbs. and horizontally from the same point the force 7793 lbs. Completing the parallelogram gives the line of resultant pressure which cuts the base at Z, Fig. No. 3, a distance of 1.4 feet outside of the middle third of the base.

To find the point where the resultant pierces the base at the joint (c^1), Fig. 3—

For horizontal pressure the formula is

$$P = x\,a\,w. \quad x = 12 \text{ feet};\ a = 9 \text{ feet};\ w = 62.5 \text{ lbs.}$$
$$P = 12 \times 9 \times 62.5 = 6750 \text{ lbs.}$$

The reverse pressure at the part would not be great enough to materially affect the result, and is therefore neglected.

For the point of application, $\bar{x} = \frac{2}{3} \frac{a^3 - b^3}{a^2 - b^2}$. $a = 15$ feet; $b = 3$ feet.

Substituting gives, $\bar{x} = \frac{2}{3} \frac{(15)^3 - (3)^3}{(15)^2 - (3)^2} = 10.33$ feet.

Therefore $a - \bar{x} = 4.67$ feet.

Measuring this distance off on a line perpendicular to the base gives the point of application.

The weight of the masonry and the water in this case $= 13320$ lbs.

The distance between the center of gravity of the masonry and the center of gravity of the water $= 6.2$ feet.

Therefore $9641\, x = 3679\, (4.67 - x)$.

$x = 1.7$ feet. Measure this distance off on the line $(7a_4)$ gives the center of gravity at (7^1).

Laying off on the vertical through w^1, Fig. 3, a distance equal to the weight 13320 lbs., and horizontally from the same point the force 6750 lbs. Completing the parallelogram gives the line of pressure which cuts the base at (w), Fig. 3, at a distance 1.3 feet outside of the middle third of the base.

To find the point where the resultant pressure cuts the base at (d^1), Fig. 3. For the horizontal pressure, $P = x\, a\, w$, $x = 10.5$ feet; $a = 8.25$ feet; $w = 62.5$ lbs.

Therefore $P = 10.5 \times 8.25 \times 62.5 = 5414$ lbs.

For the point of application $\bar{x} = \frac{2}{3}\,\frac{a^3 - b^3}{a^2 - b^2}$ $a = 13.5$ feet; $b = 3$ feet.

Substituting in the formula gives $x = \frac{2}{3}\,\frac{13.5^3 - 3^3}{13.5^2 - 3^2} = 9.34$.

Therefore $a - \bar{x} = 4.16$ which gives the point of application at (w^1), Fig. 3. The weight of the masonry is 7841 lbs., the weight of the water $= 3046$. Their sum $= 10887$. The distance between their center of gravity $= 5.8$ feet.

Therefore $7841\, x = 3046\, (5.8 - x)$.

$x = 1.62$ feet. This gives the center of gravity at ( ).

Laying off, by scale, on the vertical line through S^1, Fig. No. 3, a distance equal to the weight 10887 lbs. and horizontally, through the same point the force 5415. Completing the parallelogram gives the line of resultant pressure which cuts the base at (3^1), Fig. 3, a distance of 1.4 feet outside the middle third of the base.

To find the point at which the line of resultant pressure cuts the base at (e^1), Fig. 3.

The formula for horizontal pressure is $P = x\, a\, w$ $x = 9$ feet; $a = 7.5$ feet; $w = 62.5$ lbs.

Therefore $P = 9 \times 7.5 \times 62.5 = 4218$ lbs.

The point of application, $\bar{x} = \frac{2}{3}\,\frac{a^3 - b^3}{a^2 - b^2}$ $a = 12$ feet; $b = 3$ feet.

Therefore $\bar{x} = \frac{2}{3}\,\frac{(12)^3 - (3)^3}{(12)^2 - (3)^2} = 8.4$ feet. $a - \bar{x} = 3.6$ feet, which gives the point of application at (S^{11}), Fig. 3. The distance between the center of gravity of the masonry and water is equal to 5.2 feet. The weight of the masonry and water is 6210 lbs. + 2484 lbs. = 8694 lbs.

Therefore $6210\, x = 2484\, (5.2 - x)$.

$x = 1.48$ feet, which gives the center of gravity at (6^1).

Lay off, by scale, from v^1, Fig. 3, vertically downward the weight 8644 lbs. and horizontally from same point the force 4218 lbs. Completing the parallelogram gives the resultant line of pressure which cuts the base at (w), a distance of 1 foot from the outside of the middle third.

To find the point where the resultant line of pressure cuts the base at (f^1), Fig. 3.

The formula for the horizontal pressure $P = x\,a\,w$; $x = 7.5$ feet; $a = 6.75$ feet; $w = 62.5$ lbs.

Therefore $P = 7.5 \times 6.75 \times 62.5 = 3164$ lbs.

The formula for the point of application is $\bar{x} = \frac{2}{3}\,\frac{a^3 - b^3}{a^2 - b^2}$ $a = 10.5$ feet; $b = 3$ feet.

Therefore $\bar{x} = \frac{2}{3}\,\frac{(10.5)^3 - (3)^3}{(10.5)^2 - (3)^2} = 7.45$ feet. Therefore $a - \bar{x} = 3.05$ feet, which will give the point of application at (w^1). The weight of the masonry is equal to 4748; the weight of the water is equal to 1992 lbs. The distance between the center of gravity of the masonry and water is equal to 4.75 feet.

Taking moments:

Therefore $4748\,x = 1992\,(4.75 - x)$.

$x = 1.4$ feet, which gives the center of gravity at (5^1), Fig. 3.

Lay off, by scale, on a vertical line through w^1 the weight 6740 lbs. and horizontally the force 3164 lbs. Completing the parallelogram gives the resultant line of pressure which cuts the base at .8 of a foot from the center third. This operation is carried far enough and in no case did the resultant pressure cut the base within its middle third.

The formula for calculating the frictional stability of the dam is, according to Fanning,

$S = (w + e - c,\ z.)\ c.$

$S =$ the frictional stability.

$w =$ the weight of the masonry.

$e =$ the downward pressure of the water.

$z =$ the maximum upward pressure of the water.

$c =$ the ratio of effective upward pressure to the maximum.

$c =$ the co-efficient of friction of the section on its bed.

Let $\theta =$ the co-efficient of frictional stability.

Neglecting the upward pressure of the water and going on the supposition that the water would not penetrate the base gives formula,—

$$S = (w + e)\ c.$$

(S) should at least be equal to 1.5 times the horizontal pressure in this case $(w + e) = 28246$ lbs.

The horizontal pressure $= 14859$ lbs.

Therefore $14859\ \theta = 28246$.

$\theta = 1.15$.

Proceeding in like manner gives the frictional stability at the different joints above the foundation.

For (S) at (a^1), Fig. 3, we have

$$(w+e) = 13747 + 5155 = 18902 \text{ lbs.}$$

$$S = 18902 \times .6 = 11341.$$

The pressure of water (horizontal) = 9075 lbs.

$$\theta\ 9075 = 11341.$$

$$\theta = 1.25.$$

For (S) at joint (b^1) we have, $(w+e) = 11610 + 4382 = 15992$ lbs.
The horizontal water-pressure = 7793 lbs.

$$S = 15992 \times .6 = 9595.$$

Therefore $\theta\ 7793 = 9595.$

$$\theta = 1.23.$$

For (S) at the joint (c^1), Fig. 3,

$$(w+e) = 9641 + 3679 = 13320 \text{ lbs.}$$

$$S = 13320 \times .6 = 7992 \text{ lbs.}$$

The horizontal pressure of water = 6750 lbs.
Therefore $6750\ \theta = 7992.$

$$\theta = 1.18.$$

For (S) at the joint (d^1), Fig. 3,

$$(w+e) = 7841 + 3046 = 10887 \text{ lbs.}$$

$$S = 10887 \times .6 = 6532.$$

The horizontal water-pressure = 5144 lbs.
Therefore $5144\ \theta = 6532.$

$$\theta = 1.2.$$

For (S) at the joint (e^1), Fig. 3,

$$(w+e) = 6210 + 2484 = 8694 \text{ lbs.}$$

$$S = 8694 \times .6 = 5216.$$

Horizontal pressure = 4218.
Therefore $\theta\ 4218 = 5216.$

$$\theta = 1.23.$$

For (S) at joint (f^1), Fig. 3,

$$(w+e) = 4748 + 1992 = 6740 \text{ lbs.}$$

$$S = 6740 \times .6 = 4044 \text{ lbs.}$$

Horizontal water-pressure = 3164 lbs.
Therefore $\theta\ 3164 = 4044.$

$$\theta = 1.27.$$

This is far enough to carry the operation to determine whether the dam is sufficiently stable, and in no case has the factor been great enough to insure safety.

The formula for double stability is, according to Fanning,

$$A\ w\ d = 2\ x\ a\ w.\ d., \text{ or the factor of safety} = \frac{A\ w\ d}{x\ a\ w.\ d.}$$

$A\ w$ = the weight of a section 1 foot wide of the dam and water resting upon it.

d = the arm of $A\ w$.

d = the arm of $x\ a\ w$.

x, a, and w are used as in previous calculations.

The dam tends to turn about its down-stream toe A, Fig. 3, therefore we take moments about this point. Letting $x=$ the factor of safety, we have the formula—

$d=\frac{A\ w\ d}{x\ a\ w.\ d.}$ For the whole dam; $d=6.1$ feet; $d.=7.9$ feet; $A\ w=28246$ lbs.; $a\ w.\ x=14859$ lbs.

Therefore $x=\frac{28246\times 6.1}{14859\times 7.9}=1.46.$

Proceeding in like manner with the parts above the several joints gives the factor of safety for these parts. Taking moments about (a^1), Fig. 3, gave the factor of safety of the masonry on the foundation.

$x=\frac{A\ w\ d}{x\ a\ w.\ d.}$ $d=4.4$ feet; $d.=5.72$ feet; $A\ w=18902$ lbs.; $x\ a\ w.=9075$ lbs.

Substituting in formula gives, $x=\frac{18902\times 4.4}{9075\times 5.72}=1.59.$

Taking moments about (b^1), Fig. 3, gives the factor of safety of part of dam above this joint.

$x=\frac{A\ w\ d}{x\ a\ w.\ d.}$ $d=4.1$ feet; $d.=5.2$ feet; $A\ w=15992$ lbs.; $x\ a\ w.=7793.$

Substituting in formulæ gives, $x=\frac{15992\times 4.1}{7793\times 5.2}=1.6.$

Taking moments about (c^1), Fig. 3, gives the factor of safety for this part.

The formula is, $x=\frac{A\ w\ d}{x\ a\ w.\ d.}$ $d=3.75$; $d.=4.67$; $A\ w=13320$ lbs.; $x\ a\ w.=6750$ lbs.

Substituting in formula, $x=\frac{13320\times 3.75}{6750\times 4.67}=1.52.$

Taking moments about (d^1), Fig. 3, gave the factor of safety for part above this joint.

The formula is, $x=\frac{A\ w\ d}{x\ a\ w.\ d.}$ $d=3.5$ feet; $d.=4.16$ feet; $A\ w=10887$; $x\ a\ w.=5414.$

Substituting in the formula gives, $x=\frac{10887\times 3.5}{5414\times 4.16}=1.69.$

Taking moments about (e^1), Fig. 3, gives the factor of safety of part above this joint.

The formula is, $x=\frac{A\ w\ d}{x\ a\ w.\ d.}$ $d=3.1$ feet; $d.=3.6$ feet; $A\ w=8694$ lbs.; $x\ a\ w=4218$ lbs.

Substituting in the formula gives, $x=\frac{8694\times 3.1}{4218\times 3.6}=1.77.$

Engineering principles require 2 as a factor of safety, and we have seen in each case that these factors fall considerably below this figure.

In finding the centers of gravity, the graphical method was used as check to the method of moments. To find the center of gravity graphically of any two courses, join their centers by a line and lay off, by scale, on a perpendicular to this line at the center of the first course a distance equal to the weight of that course; on a perpendicular to the center of gravity of the second course lay off, in an opposite direction, a distance equal to the weight of the second course. Join these points by a line, and the place in which it intersects the line joining the given centers will be the center of gravity sought for. Fig. 1 shows the work of finding the center of gravity of the masonry. The center of the whole mass of masonry was found to be at (9). Fig. 2 shows the work of finding the center of gravity of the water resting on the section of dam. This is at (9).

Both methods gave the center of gravity at the same point.

In regard to profiles, Fanning says: "It is very evident that the profile has an important influence upon the leverage stability of a wall of given weight of material. The leverage stability against pressures of water upon the vertical sides of triangular or trapezoidal sections of masonry is greater than the leverage resistances to pressures upon their inclined sides; hence there is an advantage in giving all the batter to the side *opposite* to the pressure."

In the case of this dam the batter was given to the side toward the pressure.

The buttresses evidently slightly strengthen the dam at the points where they are placed, but have no material effect upon the strength of the whole structure.

To sum up, we would say:

1st. The profile of the dam is directly opposed to profiles as given by best engineers.

2d. The co-efficient of stability is too small.

3d. The factors of safety are not sufficient to insure stability.

4th. The points of application of the resultant pressures do not fall within the center third of the base. Although such a dam may stand for many years, yet as an engineering structure, it fails to meet the essential requisites of a good dam.